On fine structure emission associated with plasmaspheric density irregularities

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[1] The eigenvalue analysis for the upper-hybrid/Langmuir waves trapped in cylindrical density structures is carried out. Under physical parameters typical for kilometric continuum (KC) source region, it is found that the frequency spacing between individual fine structure spectrum is within a reasonable observed range of 0.1∼1 kHz. It is also found that the quasi field-aligned modes are associated with density cavities, while the radial modes are excited within enhanced density column, thus showing that density irregularities of various types can support discrete wave spectrum featuring wide-ranging frequency gaps. **Citation:** Yoon, P. H., and J. D. Menietti (2005), On fine structure emission associated with plasmaspheric density irregularities, *Geophys. Res. Lett.*, *32*, L23103, doi:10.1029/2005GL023795.

1. Introduction

[2] *Hashimoto et al.* [1999] described kilometric continuum (KC) as emission that lies in the same frequency range as auroral kilometric radiation (AKR) [cf. *Gurnett*, 1974], but with a different spectral morphology and a unique source region, the low-latitude, inner plasmasphere. KC was observed by Geotail (in an equatorial orbit) to consist of slowly drifting narrowband signals in the frequency range 100 kHz $< f <$ 800 kHz. Direction-finding using spin modulation of the emission at 400 kHz indicated that the source is consistent with the low-magnetic-latitude inner plasmasphere. The emission thus has much in common with non-terrestrial continuum emission [cf. *Gurnett and Shaw*, 1973; *Gurnett*, 1975] observed at lower frequencies (typically < 100 kHz).

[3] *Menietti et al.* [2003] reported high resolution observations made by the plasma wave instrument (PWI) on board the Polar spacecraft as it was passing through the source region of the KC emission observed also by Geotail. These wideband measurements indicated not just emission at f_{ce} and its (n + 1/2) harmonics, but at multiple wave frequencies separated in frequency by $f \ll f_{ce}$. The authors conclude that these fine-structure emissions were consistent with electrostatic (ES) upper-hybrid (UH) emissions as well as electromagnetic (EM) Z and O-mode waves. In addition, *Menietti et al.* [2005] discussed Polar and Cluster observations of continuum emission fine structure. High resolution waveform data from Polar were used to directly measure the polarization of continuum emission and the direction of propagation. The authors found that the continuum emission at high resolution has a fine structure very similar to KC emission. One explanation for such emission is that it is due to multiple sources located at different radial distances (hence frequency). A radial separation of <1000 km could explain differences of f_{ce} of only a few hundred Hz. It is also known that density fluctuations in the form of cavities and ducts are present in the plasmasphere [e.g., *LeDocq et al.*, 1994; *Darrouzet et al.*, 2002, 2004; *Carpenter et al.*, 2002].

[4] *Decreau et al.* [2004] observed many examples of continuum emission, some associated with electron cyclotron harmonics within the plasmasphere and near the plasmapause. The Cluster satellites encounter continuum emission source regions, which appear to be located near the magnetic equator as well as well away from the equator. *Decreau et al.* [2004] show evidence that the source regions are likely associated with density cavities. *Darrouzet et al.* [2004] conducted a statistical study of the Cluster WHISPER and EFW (electric field and waves) data to obtain a sample of the nature and extent of density irregularities observed within the plasmasphere. These authors found that the characteristic transverse equatorial size is about 365 km. The characteristic density depletion ratio is 20%.

[5] The IMAGE satellite has made recent advances in the study of small-scale field aligned density structures of the plasmasphere. *Carpenter et al.* [2002] report the results of a study of data from the Radio Plasma Imager (RPI) onboard that satellite. This instrument uses active sounding techniques from a range of frequencies. These authors conclude, based also on earlier evidence from topside sounders and whistler mode instruments [*Muldrew*, 1969] that density irregularities have cross-field scale sizes within the range of ∼200 m to over 10 km and electron densities within ∼10% of background values. *Jacobson and Erickson* [1993] used a Very Large Array radio interferometer to detect density irregularities extending 30 to 40 km transverse to the magnetic field with density fluctuations less than 10% of background. These irregularities were observed within the plasmasphere for $L < 2$. *Calvert and Warnock* [1969] used topside sounder data and "combination mode ducting" as evidence for density irregularities elongated in the direction of the geomagnetic field. *Fung et al.* [2003] have reported RPI observations of IMAGE for the outer plasmasphere (300–600 kHz) and plasmapause (100–200 kHz) that support a range of cross-field scales for density irregularities in the range ∼0.25–2 km, consistent with the work of

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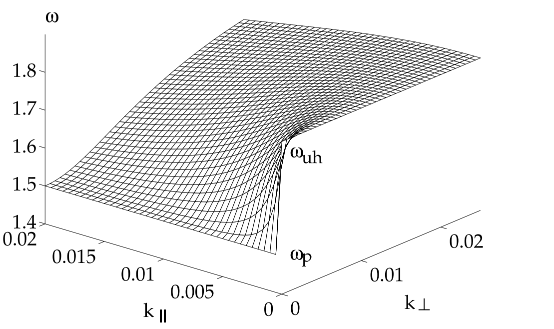

Figure 1. Upper-hybrid/Langmuir dispersion relation ω/Ω surface versus $k_\perp\, v_T/\sqrt{2}\,\Omega$ and $k_\parallel\, v_T/\sqrt{2}\,\Omega$.

Carpenter et al. [2002]. Eigenmode trapping of *Z*-mode emissions for both density cavities and density enhancements [cf. *McAdams et al.*, 2000; *Yoon et al.*, 2000] has been suggested to explain discrete emissions (fine structure) of auroral roar. A modification of such processes might also explain the plasmaspheric observations of continuum and kilometric continuum.

2. Trapped Eigenmode Theory in Density Irregularities

[6] We assume that uniform magnetic field is along the z axis while the density gradient is in the radial direction. Quasi ES waves trapped in the density structure are assumed to have wave frequency near the local upper-hybrid frequency, $\omega \simeq \omega_{\rm uh} = (\omega_p^2 + \Omega^2)^{1/2}$, or plasma frequency $\omega \simeq \omega_p$, where $\omega_p^2 = 4\pi n(r)\, e^2/m$ and $\Omega = eB/mc$. Conversion to O mode radiation is necessary for these waves to be detected from a distance.

[7] The nonlocal wave equation can be obtained from Vlasov-Maxwell linear theory. It is possible to derive the following coupled wave equation – derivation is omitted for lack of space, but full discussion will follow in a regular-length article:

$$\begin{aligned} 0 &= \frac{v_T^2}{2\Omega^2}\frac{\omega_p^2}{\Omega^2}\,\nabla_\perp^4\phi - \frac{\omega^2-\omega_{\rm uh}^2}{\omega^2}\,\nabla_\perp^2\phi \\ &\quad + k_\parallel^2\,\frac{\left(\omega^2-\Omega^2\right)\left(\omega^2-\omega_p^2\right)}{\omega^4}\,\phi, \\ 0 &= \nabla_\perp^2 A + \frac{\omega^2-\omega_p^2-c^2k_\parallel^2}{c^2}\,A \\ &\quad - k_\parallel\,\frac{\omega^2-\omega_p^2}{c\,\omega}\,\phi, \end{aligned} \tag{1}$$

where ϕ is the wave electric potential associated with the trapped ES mode, and A is the z-component of the vector potential which represents the O-mode radiation. (1) shows that once ES eigenfunction ϕ is obtained, the EM mode A is automatically generated. Note that O-mode couples to ES mode when $k_\parallel$ is finite. Mode-converted O-mode is expected to be generated in an oblique direction with respect to the density gradient. In what follows, we focus on the ES wave equation only. Full analysis of the entire coupled mode equation (1) is the subject of a future full-length paper.

[8] In the local approximation ($\nabla_\perp \to ik_\perp$), the first equation in (1) is an algebraic equation. Figure 1 shows the local dispersion relation for $\omega_p/\Omega = 1.5$. Note that for quasi-perpendicular propagation the waves can be characterized as the upper-hybrid waves, while the for quasi-parallel propagation the waves behave in a Langmuir wave-like manner. In what follows, we shall thus consider the above two extreme cases.

[9] For quasi-parallel propagation, where we may ignore the highest-order derivative and assume that $\phi(\mathbf{r}) = \phi(r)\exp(im\varphi)$, $m = 0, 1, 2, \cdots$, where φ represents the azimuthal angle. Then we have

$$\frac{1}{r}\frac{d}{dr}\,r\,\frac{d\phi}{dr} - \frac{m^2}{r^2}\,\phi - k_\parallel^2\,\frac{\left(\omega^2-\Omega^2\right)\left(\omega^2-\omega_p^2\right)}{\omega^2\left(\omega^2-\omega_{\rm uh}^2\right)}\,\phi = 0. \tag{2}$$

Let us denote the density at the axis of the cylindrical density structure by $n(0)$. In the vicinity of the axis, we may approximate $n(r) \approx n(0) + n''(0)\, r^2/2$. This leads to

$$\begin{aligned} 0 &= \frac{1}{r}\frac{d}{dr}\,r\,\frac{d\phi}{dr} + k_0^2\left(1-\frac{r^2}{r_0^2}\right)\phi - \frac{m^2}{r^2}\,\phi, \\ k_0^2 &= k_\parallel^2\,\frac{\omega^2-\Omega^2}{\omega^2}\,\frac{\omega^2-\omega_p^2(0)}{\omega_p^2(0)+\Omega^2-\omega^2}, \\ r_0^2 &= \frac{2n(0)}{n''(0)}\,\frac{\omega^2-\omega_p^2(0)}{\omega_p^2(0)}. \end{aligned} \tag{3}$$

[10] To obtain the discrete spectrum, we employ the eikonal matching condition [*McAdams et al.*, 2000; *Yoon et al.*, 2000],

$$\left(j+\frac{1}{2}\right)\pi = k_0\int_{r_-}^{r_+} dr\,\left(1-\frac{r2}{r_0 2}-\frac{m2}{k_0 2r2}\right)^{1/2},$$

where $r_\pm^2/r_0^2 = [1 \pm (1 - 4m^2/k_0^2\, r_0^2)^{1/2}]/2$, and $j = 0, 1, 2, \cdots$. Carrying out the integration, we obtain

$$k_0\, r_0 = 8\,(\,j\ +\ m/4\ +\ 1/2),$$

from which we obtain an approximate expression for the discrete eigenfrequency,

$$\begin{aligned} \omega_{j,m} &\approx \omega_p(0) + (j+m/4+1/2)\,\Delta\omega, \\ \Delta\omega &= \frac{\omega_p(0)}{k_\parallel}\sqrt{\frac{\Omega^2}{\omega_p^2(0)-\Omega^2}\,\frac{n''(0)}{n(0)}}. \end{aligned} \tag{4}$$

The quantity of interest is the frequency spacing between individual fine structure spectrum, $\Delta\omega$. For $\omega_p^2(0) > \Omega^2$, which is valid for the plasmasphere, only the density depletion $n''(0)/n(0) > 0$ will support the discrete wave spectrum. In the opposite case, namely, $\omega_p^2(0) < \Omega^2$, it will be the enhanced density column [$n''(0)/n(0) < 0$] which supports discrete trapped eigenmodes instead.

[11] Let us assume that plasmaspheric electron temperature is $T_e \sim 1$ eV, the number density is roughly $n_e \sim 50$ per c.c., and that the ratio of electron plasma frequency to gyrofrequency is on the order of $\sim$10 or so. If we assume that the free energy source of the discrete waves is a field-

aligned energetic electron beam with typical average beam speed roughly ten times the thermal speed $V_0/v_T \sim 10$, and that the characteristic wave number associated with the waves excited by such a beam satisfies the Landau resonance condition $k_\parallel\ V_0/\omega_p(0) \approx 1$, then we find that the order of magnitude estimate of the frequency spacing is $\Delta\omega/\omega_p(0) \approx 10^{-5}\ v_T/(\omega_p\ L) \propto 10^{-3}\ L^{-1}$, where L is the characteristic width associated with the density cavity measured in km's. If we assume that the plasma frequency $f_p(0) = \omega_p(0)/(2\pi)$ is on the order of 100 kHz or so, then for 1 km density structure width, we have the frequency width $\Delta f = \Delta\omega/(2\pi) \sim 0.1$ kHz. Clearly, wider density structure will lead to even narrower frequency spacing and vice versa.

[12] In the case of quasi-perpendicular wave propagation, we may ignore $k_\parallel$ in the first equation of (1):

$$0 = \frac{1}{r}\frac{d}{dr}\, r\, \frac{d\phi}{dr} - \frac{m^2}{r^2}\,\phi - \frac{2\Omega^4\left(\omega^2 - \omega_{\rm uh}^2\right)}{v_T^2\,\omega_p^2\,\omega^2}\,\phi. \tag{5}$$

Upon expanding the density in Taylor series about the cylinder axis as before, (5) can be shown to reduce to the same form as (3), except that the quantities k_0 and r_0 are now defined by

$$k_0^2 = \frac{2\Omega^4\left[\omega_p^2(0) + \Omega^2 - \omega^2\right]}{v_T^2\,\omega_p^2(0)\,\omega^2},$$
$$r_0^2 = \frac{2n(0)}{-n''(0)}\,\frac{\omega_p^2(0) + \Omega^2 - \omega^2}{\omega_p^2(0)}. \tag{6}$$

The resulting discrete upper-hybrid wave dispersion relation is approximately given by

$$\omega_{j,m} \approx \omega_{\rm uh}(0) - (j + m/4 + 1/2)\,\Delta\omega,$$
$$\Delta\omega = 2v_T\,\frac{\omega_p^2(0)}{\Omega^2}\,\sqrt{\frac{-n''(0)}{n(0)}}. \tag{7}$$

According to (7), for the upper-hybrid waves to be trapped in cylindrical density structure, it must feature an enhanced density along the density tube. A rough estimate of $\Delta\omega$ making use of the same physical parameters as before shows that in the case of enhanced density column $\Delta\omega/\omega_{\rm uh}(0) \propto 10^{-1}\ L^{-1}$. This shows that for 100 kHz upper-hybrid mode frequency, the density structure of 10 km radius will give $\sim$1 kHz frequency spacing.

3. Conclusions

[13] To conclude, in this article we have carried out the eigenvalue analysis for the upper-hybrid/Langmuir waves trapped in cylindrical density structures. The method of the analysis is similar to that considered by *McAdams et al.* [2000], who emphasized the quasi-parallel trapped Langmuir waves, and by *Yoon et al.* [2000], who paid special attention to upper-hybrid waves trapped in density enhancement. In the present analysis, we have provided a unified formalism which encompasses both situations.

[14] The works by *McAdams et al.* [2000] and by *Yoon et al.* [2000] were meant for lower-ionospheric auroral region, which is very different in terms of physical parameters that characterize the plasmapause and the source of KC emission. Thus, it is not at all intuitively clear whether the theory developed for the lower auroral ionosphere is immediately applicable to the plasmaspheric density irregularities. In the present analysis, making use of physical parameters typical for KC emission source region, we found that not only is the eigenmode trapping theory applicable, but also the frequency spacing between individual fine structure spectrum appears to be within a reasonable observed range.

[15] *Menietti et al.* [2003] have reported observations of kilometric continuum fine structure bands (see their Figure 10). Using spectral analysis of the data for their Figure 10, we have determined that the width of a spectral line is about 50–100 Hz and the spacing between the lines is about 300 Hz. Assuming an electron temperature of 1.0 eV [cf. *Rymer et al.*, 2001], number density at center of cavity $\sim$340 cm^{-3}, f_{ce} = 80 kHz [*Menietti et al.*, 2003], $V_0/v_T = 10$, and a Landau resonance condition, equation (4) yields $L \sim 1.3$ km. This is well within the range of values estimated for density irregularities observed by IMAGE RPI [cf. *Carpenter et al.*, 2002; *Fung et al.*, 2003]. We conclude that the wave fine structure may be a remote diagnostic tool for detecting small scale density fluctuations perhaps embedded within larger scale structures.

[16] Finally, we find that the quasi-parallel Langmuir/upper-hybrid modes possess generally narrower frequency spacing, and are associated with density cavities, while the quasi-perpendicular (i.e., radial) modes typically possess wider frequency gaps, and are associated with an enhanced density column.

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