Direction finding with Whisper on Cluster: some work done at Sussex

- Multipoint possibilities
  - “Triangulation”
    - => volume finding
  - Measurement of polarisation state
- High time resolution of Whisper normal mode
  - Hilbert transform
  - Kalman filtering

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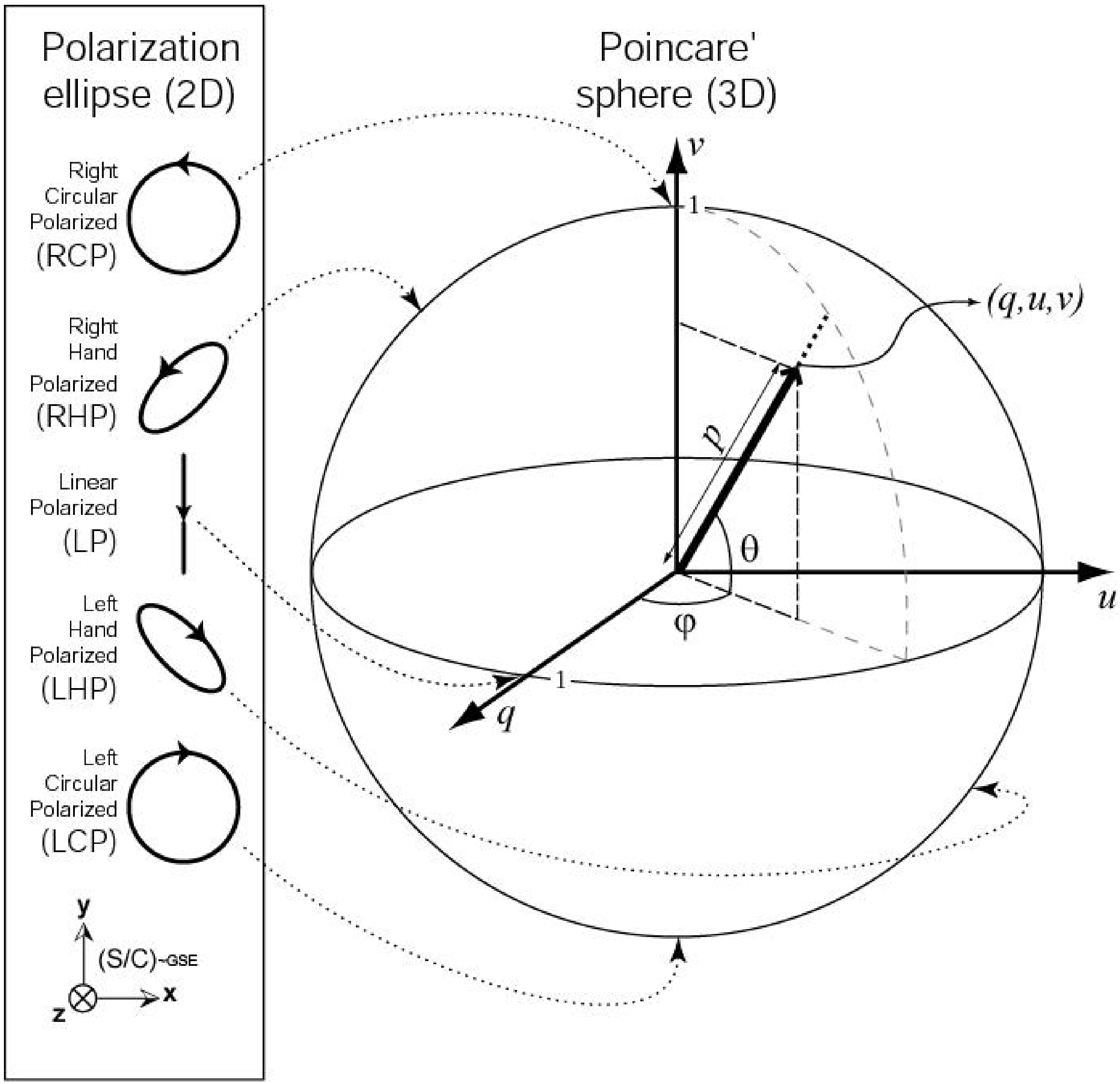
Polarisation effects: review



Modelling polarisation in the spin plane of a spinning dipole

Dipole antenna modelled by effective length vector (GSE) parametrised by S/C spin phase

$$\ell = \begin{bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{bmatrix}$$

transverse polarisation state tensor

$$\rho_T = \begin{bmatrix} I+Q & U-iV & 0 \\ U+iV & I-Q & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Rotate through elevation angle and then azimuth angle

$$\rho_{SC} = R_{az} R_{el} \rho_T R_{el}^{\mathsf{T}} R_{az}^{\mathsf{T}}$$

where

$$R_{el} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & \sin\psi \\ 0 & -\sin\psi & \cos\psi \end{bmatrix} \quad R_{az} = \begin{bmatrix} \cos\delta & \sin\delta & 0 \\ -\sin\delta & \cos\delta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The power as measured by the antenna system

$$P_\ell = \ell^\dagger \rho_{SC} \ell = A\cos 2\theta + B\sin 2\theta + C$$

where

$$A = \left(\frac{1}{2}\cos^2\psi + \cos^2\delta - \frac{1}{2} - \cos^2\psi\cos^2\delta\right) I + \left(-\frac{1}{2} + \cos^2\delta - \frac{1}{2}\cos^2\psi + \cos^2\psi\cos^2\delta\right) Q + (2\cos\delta\sin\delta\,\mathrm{co}$$

$$B = (-\cos\delta\sin\delta + \cos^2\psi\sin\delta\cos\delta)\, I + (-\cos\delta\sin\delta - \cos^2\psi\sin\delta\cos\delta)\, Q + (2\cos^2\delta\cos\psi - \cos\psi)\, U$$

$$C = \frac{1}{2}\cos^2(\psi) I - \frac{1}{2}\cos^2(\psi) Q + \frac{1}{2} I + \frac{1}{2} Q$$

This is system of 3 equations but with 5 unknowns => underdetermined!

To proceed we assume axial symmetry so Q=U=0 (even though it is known that NTC is ~60% elliptically polarised!)

In ANY case, we find that it is not possible to measure the ellipticity (V) and especially the handedness (V>0 or V<0) of the polarisation

$$P_\ell = \ell^\dagger \rho_{sp} \ell = I_{sp} + Q_{sp}\cos 2\theta + U_{sp}\sin 2\theta$$

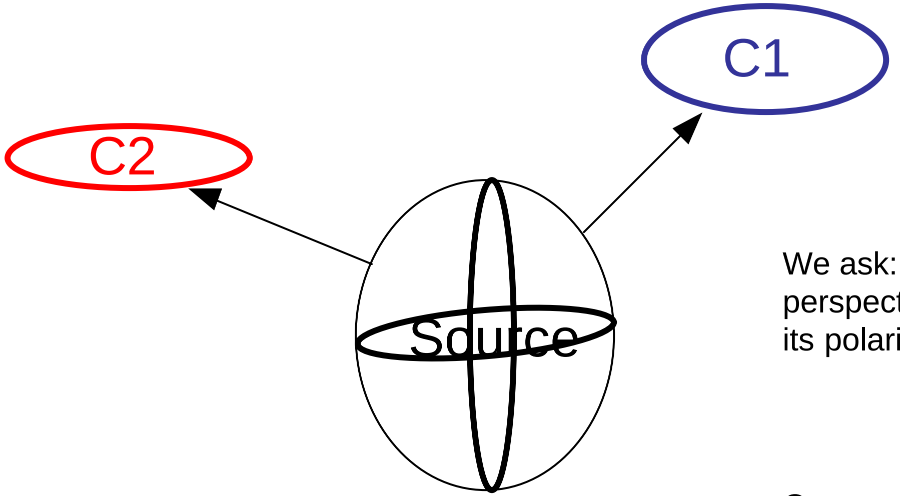
$$p^2 I^2 = Q^2 + U^2 + V^2$$

$$v_{sp} := \frac{V_{sp}}{I_{sp}} = 1 - \frac{\sqrt{Q_{sp}^2 + U_{sp}^2}}{I_{sp}} = 1 - m$$

Prospects for multipoint polarisation measurement?

In the general case (extended source region, arbitrary polarisation etc.) the second order description of transverse EM radiation at a spacecraft is given by a 3D rank 2 polarisation tensor of which only the spin components are accessible

$$\rho_{sp} = \begin{bmatrix} I_{sp} + Q_{sp} & U_{sp} - iV_{sp} & ? \\ U_{sp} + iV_{sp} & I_{sp} - Q_{sp} & ? \\ ? & ? & ? \end{bmatrix}$$



We ask: is it possible from multiple perspectives on the source region to discern its polarisation?

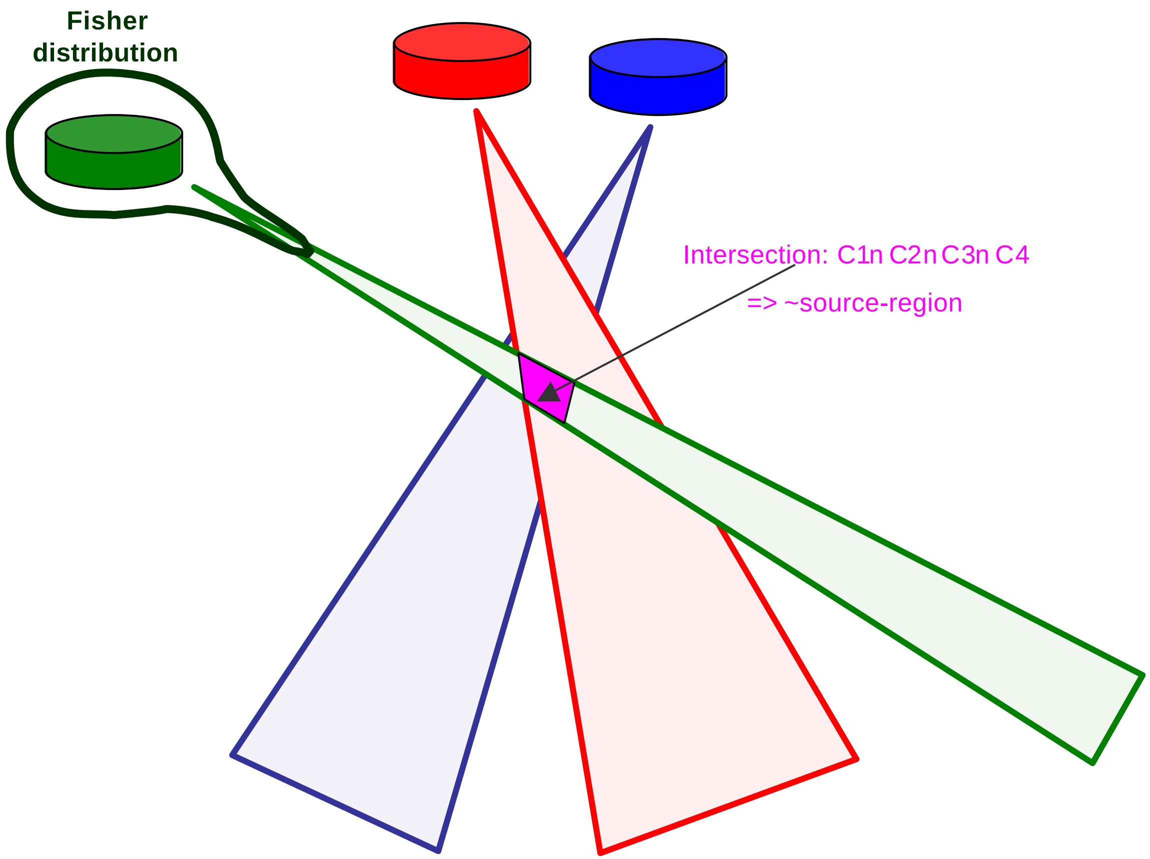
Our conclusion: this is NOT feasible

Volume finding

Since the inter-spacecraft separation is often small on Cluster in comparison with the range to source region "triangulation" based on standard triangulation is difficult

Therefore we have tried to extend to notion of direction finding to VOLUME finding

To this end we attempt to estimate the variance of DF in azimuth and elevation based on the Fisher distribution. This defines elliptical cones the intersection of which is an estimate of the "source region"



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Hi-Res Direction Finding (whisper normal mode)

- Attempt to achieve temporal resolution approaching spin period
- Requires good signal processing
- High S/N, obtained by
  - Integration over frequency bands
  - Frequency tracking algorithms
- Techniques used:
  - Windowed Least Squares Fit
  - Complex demodulation
  - Hilbert tranform
  - Kalman filter
  - Markov modeling

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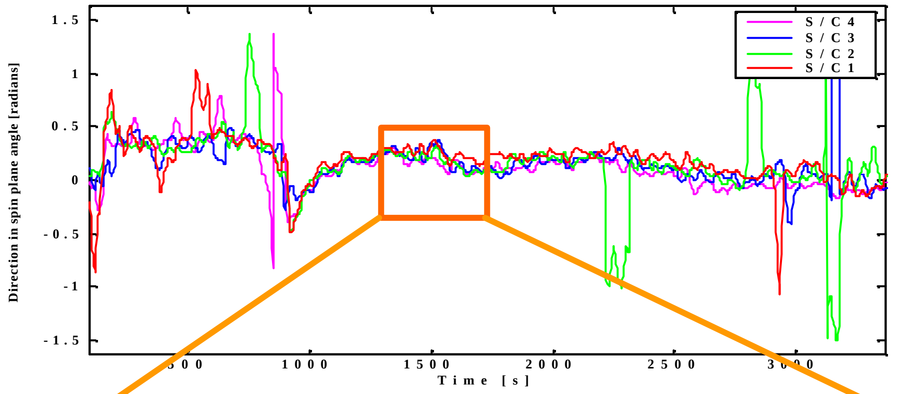
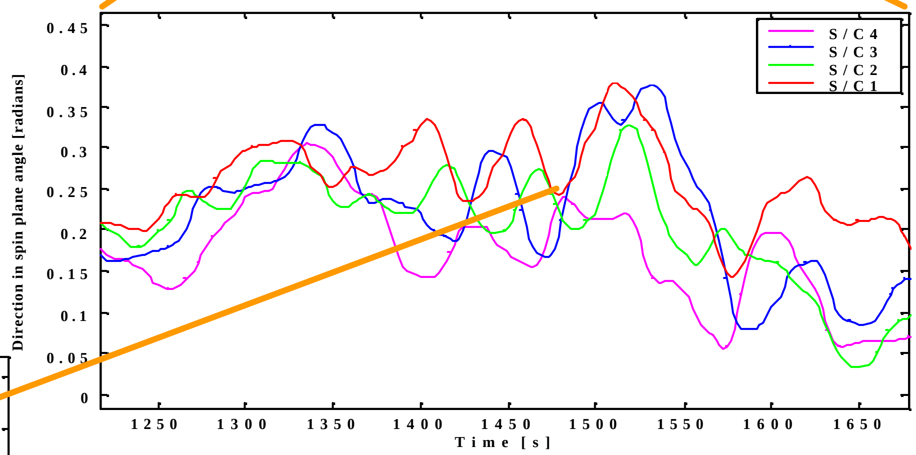
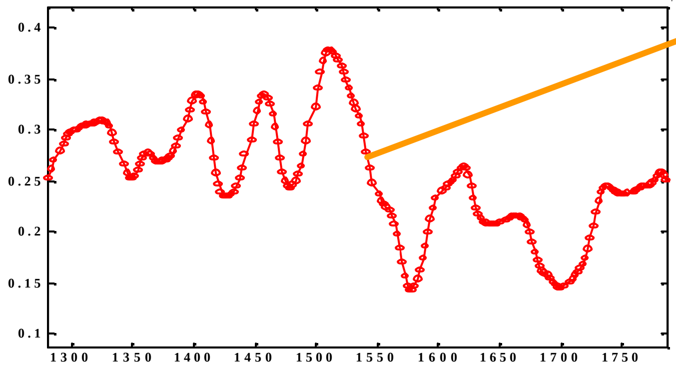
Hi-Res DF example

- Data from 2002-08-31 (same as Paul Goughs) integrated over all frequencies
- Direction of ellipse minor axis shows trend towards Earth
- Hi-res scale shows quasi continuous oscillations on 50s time scale
- Unfortunately this is due to beating with Whisper spin-sampling period
- (Has same period as Pc4?)







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