Barycentric tools for CLUSTER

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IASB, Brussels, February 24-25, 2003

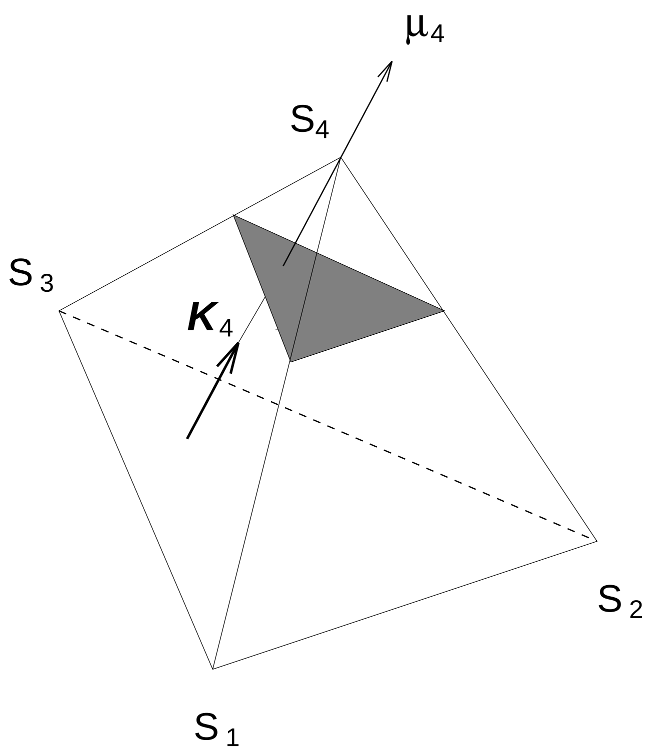
A1. Reciprocal vectors and tensor

Reciprocal vector

$$\vec{k}_4 = \frac{\vec{r}_{12} \times \vec{r}_{13}}{\vec{r}_{14} . (\vec{r}_{12} \times \vec{r}_{13})}$$

Reciprocal tensor

$$\overline{\overline{K}} = \Sigma_{\nu=1}^{4} \vec{k}_\nu \vec{k}_\nu^{\,t}$$



A2. Gradient of a vector field

Definition

$$d\vec{A}^t = d\vec{r}^{\,t}\,\vec{\nabla}\vec{A}$$

Estimator $\overline{\overline{G}}$ of $\vec{\nabla}\vec{A}$

$$\overline{\overline{G}} = \Sigma_{\alpha=1}^{4}\,\vec{k}_\alpha\,\vec{A}_\alpha^t$$

B1. Covariance of spacecraft positions

Definition

$$\langle \delta\vec{r}_\gamma \, \delta\vec{r}_\nu^{\,t} \rangle$$

Simplifying hypothesis

$$\langle \delta\vec{r}_\gamma \, \delta\vec{r}_\nu^{\,t} \rangle = \delta_{\gamma,\nu} \, \langle \delta\vec{r} \, \delta\vec{r}^{\,t} \rangle$$

B2. Covariance of reciprocal vectors

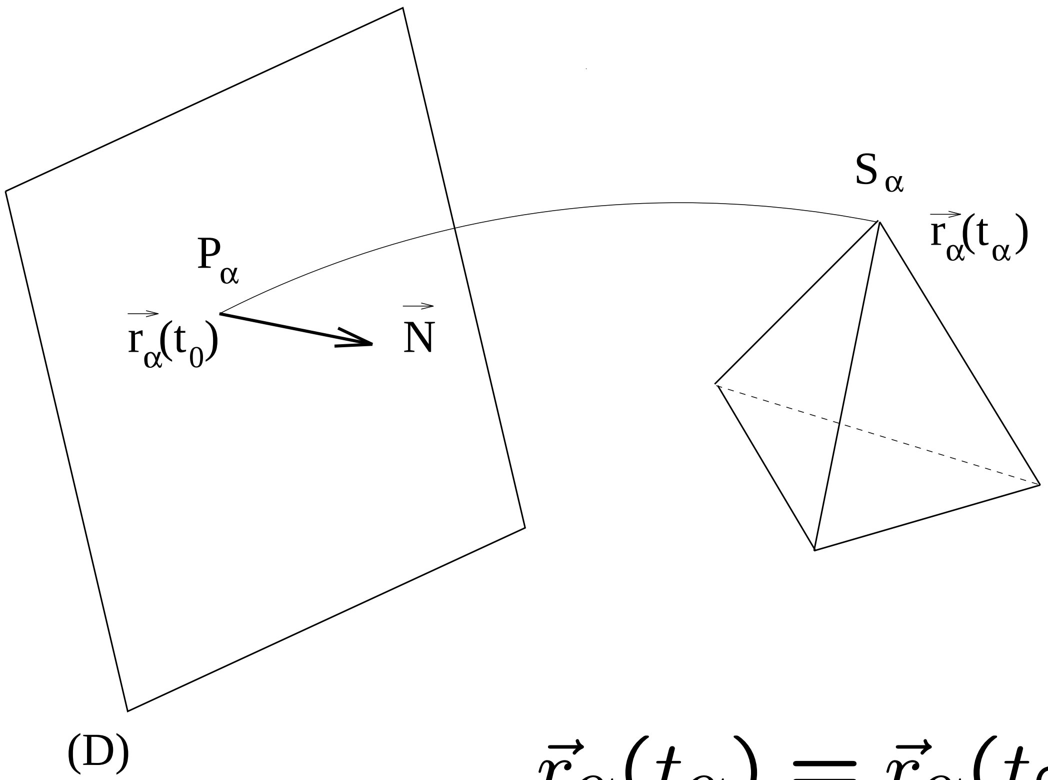
General expression

$$\langle \delta k_{\alpha i}\, \delta k_{\beta j} \rangle = \Sigma_{\gamma=1}^{4}\, \Sigma_{\nu=1}^{4}\, \vec{k}_{\alpha}^{\,t}\, \langle \vec{\delta r}_{\gamma}\, \vec{\delta r}_{\nu}^{\,t} \rangle\, \vec{k}_{\beta}\, k_{\gamma i} k_{\nu j}$$

Simplified expression

$$\langle \delta k_{\alpha i}\, \delta k_{\beta j} \rangle = \vec{k}_{\alpha}^{\,t}\, \langle \delta \vec{r}\, \delta \vec{r}^{\,t} \rangle\, \vec{k}_{\beta}\, K_{ij}$$

C1. Planar surface in uniform motion



$$\vec{r}_\alpha(t_\alpha) = \vec{r}_\alpha(t_0) + \vec{V}\,(t_\alpha - t_0)$$

C2. Timing normal and normal velocity

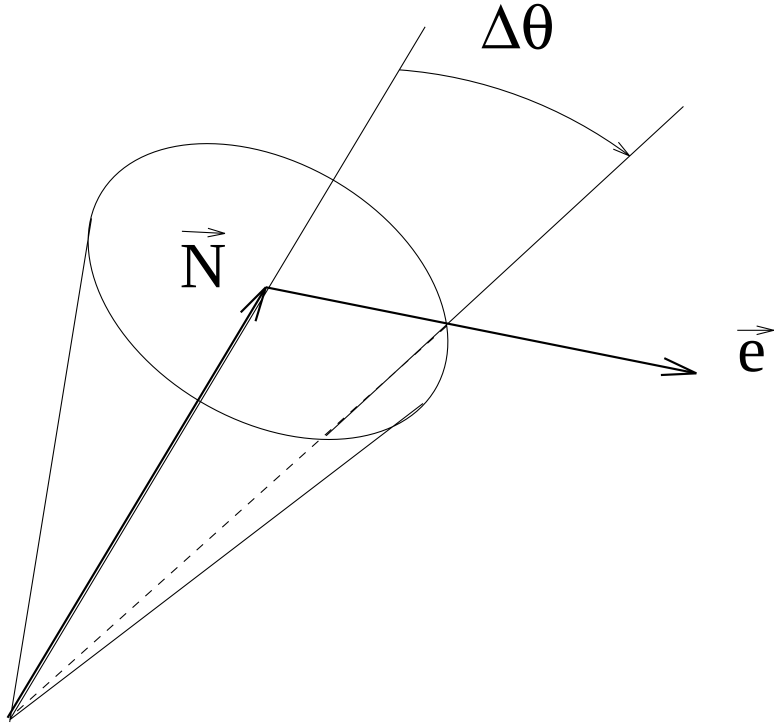
$$\begin{aligned} \vec{Q} &= \sum_{\alpha} t_{\alpha} \vec{k}_{\alpha} \\ V_n &= \left(\vec{Q} \cdot \vec{Q} \right)^{-1/2} \\ \vec{N} &= V_n \, \vec{Q} \end{aligned}$$

C3. Covariances of $\vec{N}$ and V_n

$$
\begin{aligned}
C &= (\Delta t)^2 + \vec{Q}^{\,t} \langle \delta\vec{r}\,\delta\vec{r}^{\,t} \rangle\, \vec{Q} \\
\langle \delta\vec{Q}\,\delta\vec{Q}^{\,t} \rangle &= C\, \overline{\overline{K}} \\
\langle \delta\vec{N}\,\delta\vec{N}^{t} \rangle &= C\, V_n^2 \left(\overline{\overline{I}} - \vec{N}\,\vec{N}^{\,t} \right) \overline{\overline{K}} \left(\overline{\overline{I}} - \vec{N}\,\vec{N}^{\,t} \right) \\
(\Delta V_n)^2 &= C\, V_n^4\; \vec{N}^{\,t}\, \overline{\overline{K}}\; \vec{N}
\end{aligned}
$$

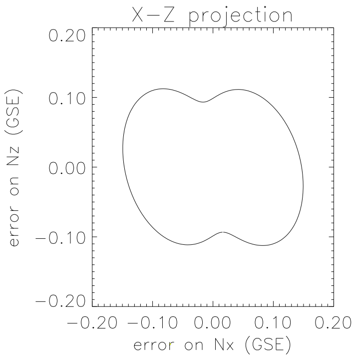
C4. Cone of uncertainty

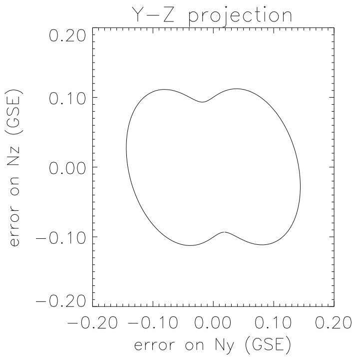
$$(\Delta\theta)^2 = \vec{e}^{\,t} \langle \delta\vec{N}\, \delta\vec{N}^t \rangle\, \vec{e}$$

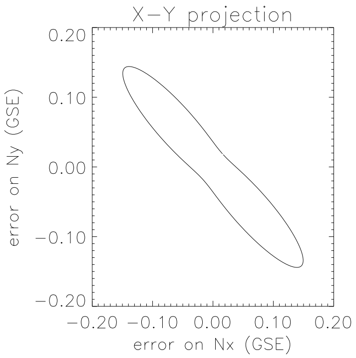


Bow shock 2000/12/22 08:20-40 UT

Earthward







GSE components of N

Nx = −0.67

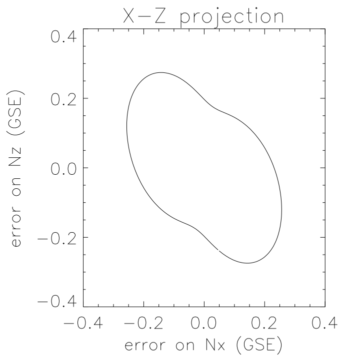
Ny = −0.70

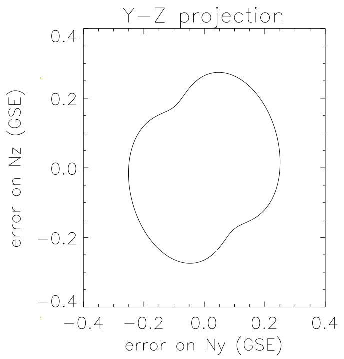
Nz = −0.25

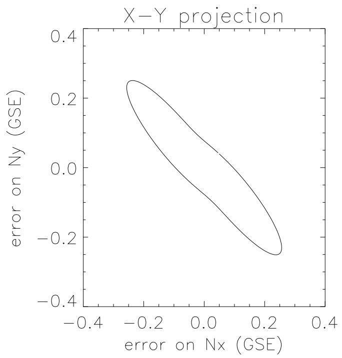
Normal velocity ($km.s^{-1}$)

5. < Vn < 11.

Sunward







GSE components of N

Nx = 0.71

Ny = 0.66

Nz = 0.26

Normal velocity ($km.s^{-1}$)

2. < Vn < 8.

D1. Curvature of field lines

$\vec{T}$ and $\vec{N}$, unit vectors, tangent and normal

$$\vec{B} = B\,\vec{T}$$

$$\frac{d}{ds}\vec{T} = \frac{\vec{N}}{\mathcal{R}}$$

$$\frac{d}{ds}(\vec{T} \times \vec{N}) = \frac{\vec{N}}{\mathcal{T}}$$

D2. Curvature of field lines

Link with the gradient of the field

$$\frac{B}{\mathcal{R}}\vec{N} = (\vec{\nabla}\vec{B})^{\,t}\vec{T} - \left[\vec{T}^{\,t}(\vec{\nabla}\vec{B})^{\,t}\vec{T}\right]\vec{T}$$

Estimator of the curvature

$$\frac{B}{\mathcal{R}}\vec{N} = \Sigma_{\alpha=1}^{4}(\vec{k}_\alpha \cdot \vec{T})\left[\vec{B}_\alpha - (\vec{T}\cdot\vec{B}_\alpha)\vec{T}\right]$$